Universal scaling in mixing correlated growth with randomness

A. Kolakowska, M. A. Novotny, and P. S. Verma
Department of Physics and Astronomy, and the ERC Center for Computational Sciences, P.O. Box 5167, Mississippi State, Mississippi 39762-5167, USA

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We study two-component growth that mixes random deposition (RD) with a correlated growth process that occurs with probability p. We find that these composite systems are in the universality class of the correlated growth process. For RD blends with either Edwards-Wilkinson or Kardar-Parisi-Zhang processes, we identify a nonuniversal exponent in the universal scaling in p.

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Many properties of complex systems can be uncovered by statistical analysis of some representative nonequilibrium interfaces. Mainstream studies of surface growth and interface roughening focus on one-component growth, or homoepitaxy, and large-scale properties. On the microscopic level, nonequilibrium interfaces have been studied in a variety of discrete simulation models such as ballistic deposition (BD), Eden, or solid-on-solid models. While one-component growths are well understood [1], the same cannot be claimed about composite systems, even as simple as binary growth in one spatial dimension. Several mixed-growth models studied in the recent decade [2,3] reveal new and nontrivial properties. The theory behind these, however, is in the initial stages.

In this systematic study of two-component growth, we examine a system whose dynamics is governed by two simultaneously present processes: one is a process that builds up correlations (a pure-correlated growth) and the other process is totally uncorrelated, i.e., random deposition (RD). The pure-correlated growth occurs with probability p. Questions that we address here concern the universality of such composite systems. As we shall show, the presence of randomness slows down the dynamics of the correlation processes. Nevertheless, the universality class of the combined processes is the same as the universality class of a correlation process. This is an outcome of scaling in p. One consequence of this observation is a magnifying-glass effect that RD blending has on the time evolution of the surface roughness. This effect can be useful in revealing hidden features of a correlated growth when designing simulation models. Intuitively, since RD carries no correlations of its own, it may be expected that its admixtures should not lead to a new universality class. Yet, demonstration of this is not so trivial since, as we shall make evident by the results of several simulations, some of the parameters involved in the universal scaling may be nonuniversal. Results presented here for $(1+1)$ dimensions are generally valid in many dimensions.

Consider aggregation models where particles fall onto a one-dimensional substrate of L sites, where they may be accepted in accordance to a rule that generates correlations among the sites. This pure-correlated growth occurs with probability p and competes with RD growth that occurs with probability $q=1-p$. When a particle is accepted at a site, the site increases its height by Δh. Roughness of the growing surface is measured by the interface width $w(t)$ at time t: $\langle w^2(t)\rangle = \langle L^{-1}\Sigma_{k=1}^{k=L}[h_k(t)-\bar{h}(t)]^2\rangle$, where $h_k(t)$ is the height at site k and $\bar{h}(t)$ is its mean over L sites (angular brackets denote the mean over N configurations).

In a pure-correlated growth ($p=1$), assuming elementary linear and nonlinear models, the self-affined roughness obeys the Family-Vicsek (FV) scaling [4],

$$w^2(t) = L^{2\alpha}F(t/L^z), \tag{1}$$

where $F(y)$ gives two evolution limits: $F(y)\sim y^{2\alpha/z}$ if $y \ll 1$ (growth); and, $F(y)\sim$ const if $y \gg 1$ (saturation). The crossover time $t_\times$ from growth to saturation is given by the dynamic exponent z, $t_\times \sim L^z$ (Fig. 1). At saturation the width does not depend on time, $w^2 \sim L^{2\alpha}$, where α is the roughness

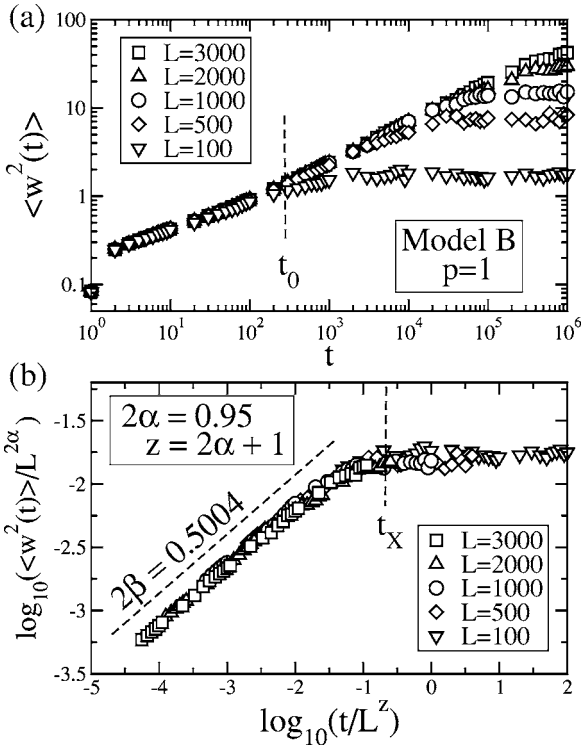

FIG. 1. Model B for pure-correlated growth: (a) time evolutions of the interface width (t_0 marks the end of the initial nonscaling regime); (b) scaling function for $t > t_0$. z, α, and β are consistent with the EW universality. Here, $N \approx 100$.

exponent. During the growth $w^2(t) \sim t^{2\beta}$, where $\beta = \alpha/z$. Exponents z, α, and β are universal. This means two different simulation models of two different correlation mechanisms will generate the same type of scaling, with consistent values of exponents, provided these mechanisms represent the same type of correlation process, i.e., belong to one universality class. Dynamics of the buildup of correlations and dynamical scaling are described within a continuum model by a stochastic growth equation. One example is the Kardar-Parisi-Zhang (KPZ) equation [5]

$$h_t = v(t) + \nu_0 h_{xx} + (\lambda_0/2) h_x^2 + \eta(x,t), \tag{2}$$

where $h = h(x,t)$ is the height field (subscripts denote partial derivatives; x is the coordinate along the substrate), v is the mean interface velocity, and η is the white noise (ν_0 and λ_0 are coefficients). In the KPZ universality class, governed by Eq. (2), $\alpha + z = 2$ and $\alpha = 1/2$. When $\lambda_0 = 0$, Eq. (2) becomes the Edwards-Wilkinson (EW) equation [6], defining the EW universality with $2\alpha + 1 = z$ and $\alpha = 1/2$. When $\nu_0 = \lambda_0 = 0$, Eq. (2) describes uncorrelated processes of RD universality, characterized by $\beta = 1/2$, $t_\times = \infty$, and the absence of scaling in L. For EW processes, Eq. (1) expresses the invariance of the EW equation under the scaling [1]

$$x \to Lx, \quad h \to L^\alpha h, \quad t \to L^z t. \tag{3}$$

Similarly, for KPZ processes it expresses the invariance of the convective derivative in the Burger's equation.

In simulations, t is the number of deposited monolayers. The first step is RD to a flat substrate. The system retains the memory of this initial condition for t_0 steps, where t_0 depends on the particulars of the model, i.e., t_0 is a nonuniversal parameter. In this start-up regime $w(t)$ does not scale [7]; scaling occurs only for $t > t_0$ (Fig. 1).

In deriving the scaling hypotheses, we are guided by the following four models: **Model A**: for $p = 1$ is RD with surface relaxation where $\Delta h = 1$ [1,8], known to be in the EW universality class (for $p < 1$, studied in [2,9]). In **Model B**, Δh is sampled from a uniform distribution of unit mean and the substrate is sampled sequentially at each t. When $p = 1$: particles that fall on the local interface minima are always accepted; particles that fall on local maxima slide down to either of the neighboring sites with probability 1/2; and, particles that fall on local slopes slide down to nearest-neighbor sites. Model B for $p = 1$ is in the EW class (Fig. 1). It simulates, e.g., deposition of a sticky nongranular material of variable droplet size. **Model C**: for $p = 1$ is BD with $\Delta h = 1$, known to be of KPZ universality [1] (for $p < 1$, studied in [2,10]). In **Model D**, Δh is sampled from a Poisson distribution of unit mean, and each monolayer is obtained by sequential sampling. When $p = 1$ in Model D, particles are deposited only to local surface minima. This case is in the KPZ universality class [7]. Model D simulates, e.g., conservative updates in a system of asynchronous processors [7,11]. We stress that, although in one universality class for $p = 1$, Models C and D are *essentially different* simulations (as is the pair A and B).

In all models, evolutions $w^2(t)$ form two-parameter families of curves (L and p being parameters) that for any $p \in (0;1]$ look like those in Fig. 1 but with

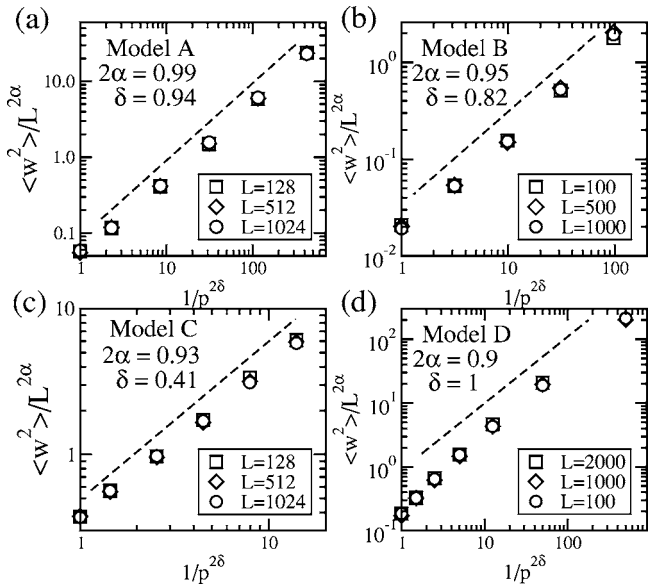

FIG. 2. Scaled widths at saturation vs parameter $1/p^{2\delta}$: (a) and (b) are for Models A and B, respectively (mix of RD with EW processes); (c) and (d) are for Models C and D, respectively (mix of RD with KPZ). Reference lines have slope 1. Data are scaled with the exponent values shown here.

$t_0 \equiv t_0(p) \geqslant t_0(1)$, $t_\times \equiv t_\times(p) \geqslant t_\times(1)$, and at saturation $w^2(p) \geqslant w^2(1)$. The curves saturate due to *only one* component, the pure-correlated deposition, since the other component, RD, introduces no correlations. At saturation, the observed lateral correlation length is $\xi_\parallel(p) \sim L$ and $t_\times(p) \sim L^z$; thus, $\xi_\parallel(p) \sim t_\times^{1/z}(p)$; and the widths scale in L as $w^2(p) \sim L^{2\alpha}$. Plots of the scaled widths $\langle w^2(p) \rangle / L^{2\alpha}$ (Fig. 2) show that they generally scale in p as $w^2 \sim L^{2\alpha}/p^{2\delta}$, where δ is some parameter. Is δ a universal exponent? Models A and B (Figs. 2(a) and 2(b) may suggest a universal value $\delta = 1$ for the RD-EW mix. But Models C and D [Figs. 2(c) and 2(d)] show that $\delta_D \approx 2\delta_C$. Accordingly, δ is not universal because for the RD-KPZ mix its value is clearly related to the technicalities of these models. In the RD-EW case there is no reason to believe that $\delta = 1$ is not accidental. Scale invariance of the EW equation is not sufficient to furnish δ.

Since $t_\times(p) \sim L^z$ and $w^2 \sim L^{2\alpha}/p^{2\delta}$ for any $p \neq 0$, the roughness must scale as $w^2(t)/w^2 \sim F[t(p)/L^z]$. This scaling in L collapses all curves $w^2(t)$ to one-parameter families (p being the only parameter now) presented in Figs. 3(a), 4(a), 5(a), and 6(a). As RD components do not build correlations, this collapse is obtained with the scaling laws from the corresponding universality classes of processes that build up correlations. Explicitly, $z = 2\alpha + 1$ and $z = 2 - \alpha$ for blending RD with EW and KPZ processes, respectively. To further collapse the data in p, i.e., to find $t(p)$ in the argument of function F, we analyze the invariance of the corresponding continuum equations under simultaneous affine transformations,

$$x \to Lx, \quad h \to hL^\alpha/g, \quad t \to tL^z/f, \tag{4}$$

assuming g and f being arbitrary suitable functions of p.

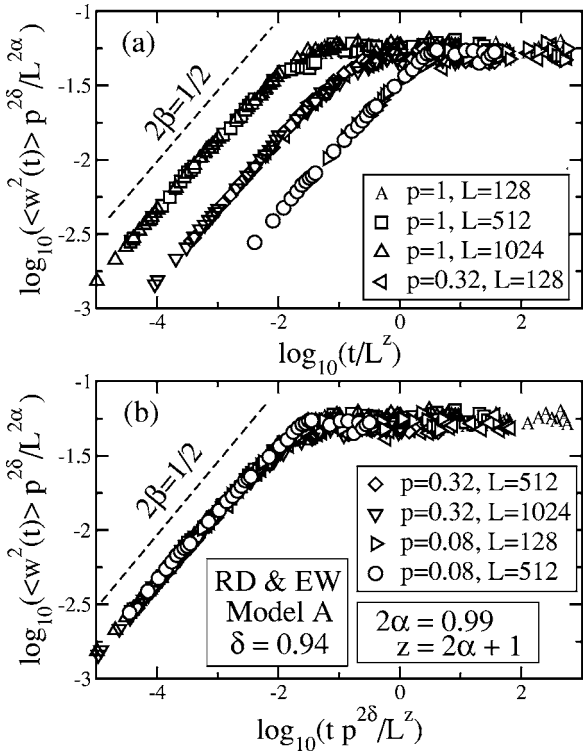

FIG. 3. Scaling for Model *A*: (a) in *L*; (b) in *p* of the data in (a). α and β are consistent with the EW universality class. Data labels are common for both figures. $N \approx 100$.

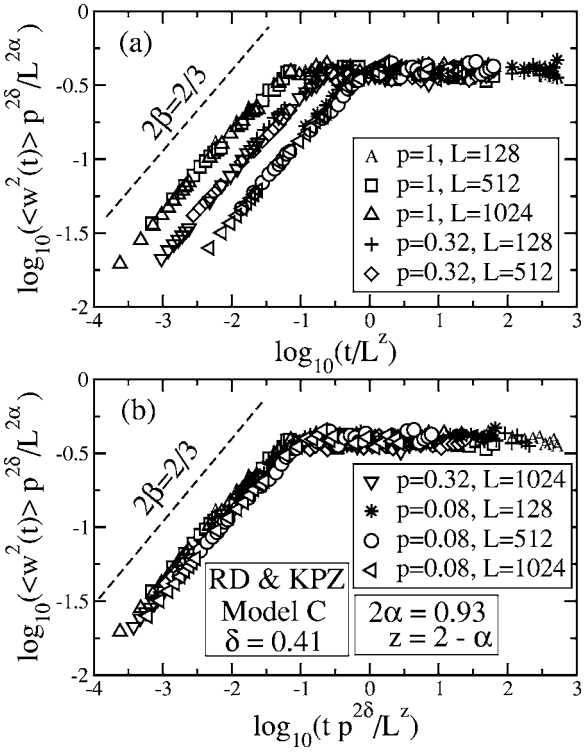

FIG. 5. Scaling for Model *C*: (a) in *L*; (b) in *p* of the data in (a). α and β are consistent with the KPZ universality. Data labels are common for both figures. Here, $N \approx 100$.

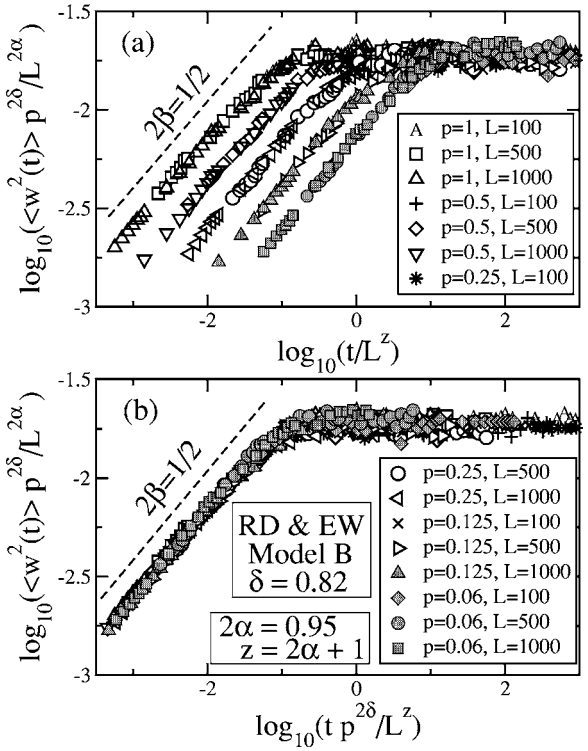

FIG. 4. Scaling for Model *B*: (a) in *L*; (b) in *p* of the data in (a). As in Model *A*, α and β indicate the EW universality class. Data labels are common for both figures. $N \approx 100$.

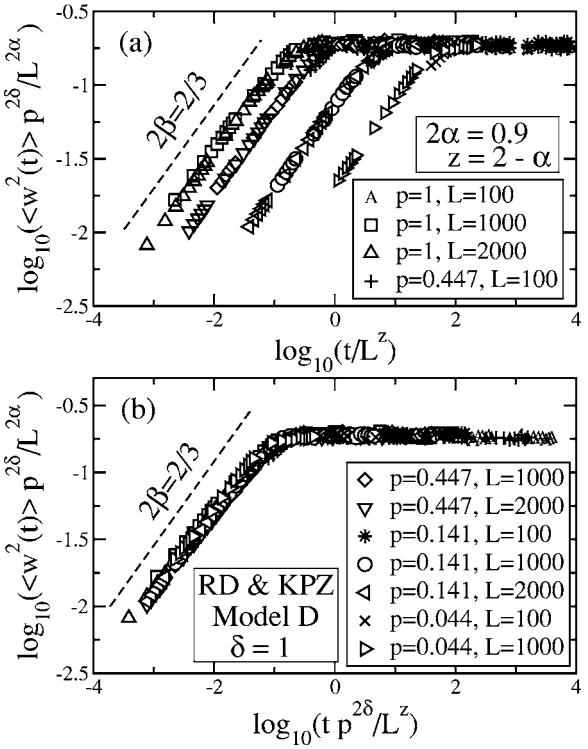

FIG. 6. Scaling for Model *D*: (a) in *L*; (b) in *p* of the data in (a). α and β indicate the KPZ universality, but $\delta_D \approx \delta_C/2$. Data labels are common for both figures. $N \approx 1000$.

Scaling (4) is the superposition of scaling (3) with

$$x \to x' = x, \quad h \to h' = h/g(p), \quad t \to t' = t/f(p). \tag{5}$$

Invariance analysis under the component scaling (3) leads to Eq. (1) and signature-scaling laws of KPZ and EW processes. This justifies the data collapse in L. The component scaling (5) transforms Eq. (2) to: $h'_{t'} = v' + \nu'(p)h'_{x'x'} + [\lambda'(p)/2]h'^2_{x'} + \eta'(x',t')$, where $h'_{t'} = h_t f/g$, $v' = vf/g$, $h'_{x'x'} = h_{xx}/g$, and $\eta'(x',t') = \sqrt{f}\eta(x,t)$. Its invariance under (5) implies

$$f(p) = g^2(p), \tag{6}$$

$$\nu'(p) = \nu_0 f(p), \tag{7}$$

$$\lambda'(p) = \lambda_0 g(p) f(p). \tag{8}$$

From scaling at saturation we obtained $g(p) = p^\delta$. Thus, the continuum equation for the RD-KPZ mix is

$$h_t = v(t) + \nu_0 p^{2\delta} h_{xx} + (\lambda_0/2) p^{3\delta} h_x^2 + \eta(x,t). \tag{9}$$

In the limits $p \to 1$ and $p \to 0$, Eq. (9) describes the dynamics of pure processes, i.e., the KPZ type and RD, respectively. Similarly, the invariance of the EW equation under scaling (5) gives Eqs. (6) and (7). This leads to the continuum equation for the RD-EW mix

$$h_t = v(t) + \nu_0 p^{2\delta} h_{xx} + \eta(x,t). \tag{10}$$

The inverse of the scaling (4) is the desired contraction that gives the full data collapse described by the FV function. The inverse of the scaling (5) alone ($x \to x$, $h \to p^\delta h$, $t \to p^{2\delta} t$) gives $t(p)$ in the argument of $F(y)$: $y = p^{2\delta} t/L^z$. Finally, the FV scaling for any two-component process, one of which is RD, is

$$w^2(t) = \frac{L^{2\alpha}}{p^{2\delta}} F\left(\frac{p^{2\delta}}{L^z} t\right), \tag{11}$$

where α and z are universal exponents of the component process that builds up correlations, and δ is nonuniversal. This result is illustrated in Figs. 3(b), 4(b), 5(b), and 6(b).

Our results, Eqs. (9)–(11), show that mixing RD with a correlated growth preserves the universality of the correlated growth. Physical justification is in the uncorrelated nature of RD. As can be seen from Eqs. (9)–(11), RD blending reduces the values of coefficients ν and λ relative to the original noise strength. In other words, the net outcome is a noisier dynamics. The analysis presented here by the examples of EW and KPZ processes in (1+1) dimensions is easily extended to other growth processes in $(1+n)$ dimensions. It is enough to notice that Eq. (6) is generally valid when scaling (5) applies to growth equations of the type $h_t(\vec{x},t) = (\text{operator})h + \eta(\vec{x},t)$, where $\vec{x}$ is n dimensional. Hence the conclusion: If a correlated growth belongs to a given universality class, its mix with RD will remain in the same class. The only effects of the RD admixture are the simultaneous dilatations of the fundamental time and height scales in accordance with scaling (5) [and Eq. (6)]. The net consequence of these is a slowdown in the dynamics of building up the correlations, reflected in the change of the lateral correlation length $\xi_\parallel(p) \sim t(p)^{1/z} = \xi_\parallel(1)/\sqrt[z]{f(p)}$. In a sense, RD blending is like applying a magnifying glass to the evolution curves $w(t)$: the smaller the p, the better the magnification. In particular, in a two-component growth that mixes RD with either EW or KPZ processes, these dilatations explicitly are $h \to h/p^\delta$ and $t \to t/p^{2\delta}$, where δ is nonuniversal and reflects the particulars of the deposition. The stretching in time causes the initial nonscaling regime $t_0(1)$ in curves $w(t)$ to be amplified as $t_0(p) = t_0(1)/p^{2\delta}$. One consequence of this amplification is a clear observation of the RD growth (with $\beta = 1/2$) for initial times $t < t_0$ when the growth starts from a flat substrate (e.g., observed in [2,9,10]). Note, if $p \to 0$ this initial phase becomes infinitely long as this is the limit of RD growth. In simulations, when p is known, by a prudent design of a model, magnifying effects of RD blending may prove advantageous in revealing hidden features of a correlated growth. However, in the laboratory, the presence of randomness in the growth process will obscure a clear-cut observation of the expected scaling.

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